A Model Predictive Control Strategy for the Control of SMB Plants using the Wave Theory

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Objective

Design a **controller** for simulated moving bed (SMB) plants based on the combination of Nonlinear Model Predictive Control (**NMPC**), model reduction through the proper orthogonal decomposition (**POD**) and the **wave theory**.

Introduction

The process description:

The SMB is a chromatographic **separation process** of paramount importance in various industrial sectors (from food to fine chemicals and pharmaceuticals).

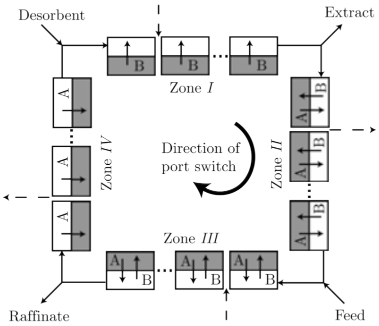
✓ The plant is divided into **four zones** (possibility of variable number of columns per zone)

✓ **Movement of the solid** is **simulated** by switching the inlet and outlet valves

✓ The **less adsorbed** component (*A*) is obtained in the **raffinate**

✓ The **stronger adsorbed** component (*B*) is obtained in the **extract**

✓ **Separation** occurs in zones *II* and *III*



The Linear Driving Force Model [1]:

Concentration in the liquid phase

$$\frac{\partial c_{kj}}{\partial t} = D_{kj}\frac{\partial^2 c_{kj}}{\partial z^2} - v_j\frac{\partial c_{kj}}{\partial z} - k_\varepsilon \frac{dq_{kj}}{dt}; \quad k_\varepsilon = \frac{1-\varepsilon}{\varepsilon}$$

Concentration in the solid phase

$$\frac{dq_{kj}}{dt} = \kappa_k (q_{kj}^{eq} - q_{kj})$$

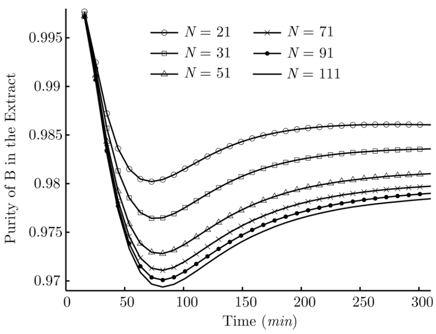
Coupling

Langmuir isotherm

$$q_{kj}^{eq} = \frac{a_k c_{kj}}{1 + \sum_{i=A,B} b_i c_{ij}}$$

Numerical simulation (FEM)

Spatial discretization (*N*) ⇒ Transform the PDE into a set of ODE



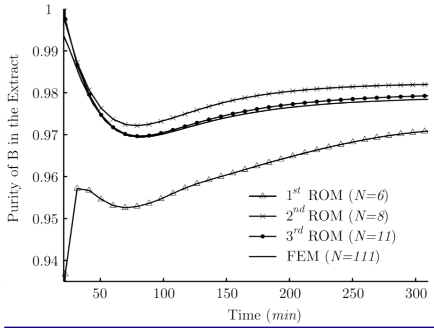
✓ Large differences between solutions with low and large *N*

✓ For a good solution, it is required at least *N=111*

✓ Computationally **too demanding for MPC**

Numerical simulation (POD)

It is based on the **time scale separation** between states. The model is reduced by **neglecting** the **fast dynamics**.



The **POD** technique in three steps [2]:

1.- Obtain **measurements** of the field. FEM simulation

2.- Compute the **basis functions** from these measurements

3.- **Project** the PDE into the basis functions

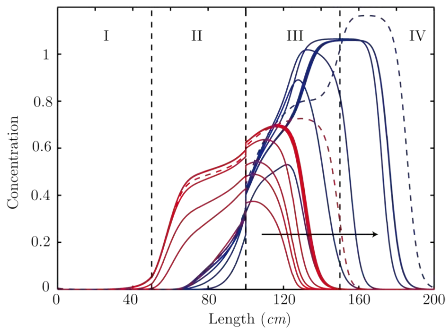
Dimensionality reduced in **more** than **one order** of magnitude

Typical concentration profiles in SMB

✓ If the working conditions are carefully chosen, good separation is obtained

✓ **Perturbations** can lead to **poor separation** (dashed lines)

✓ **Nonlinear control** techniques are required



The wave theory

The concentration profiles in SMB travel as **nonlinear waves**. Two kind of waves are found: **Spreading** waves in zones *I*, *II* and **shocks** in zones *III*, *IV*.

$$w_j = \frac{k_\varepsilon v_s \left.\frac{\Delta q_k}{\Delta c_k}\right|_{c^*} - v_j}{1 + k_\varepsilon \left.\frac{\Delta q_k}{\Delta c_k}\right|_{c^*}} \qquad w_j = \frac{k_\varepsilon v_s \left.\frac{dq_k}{dc_k}\right|_{c^*} - v_j}{1 + k_\varepsilon \left.\frac{dq_k}{dc_k}\right|_{c^*}}$$

The **purity** is **controlled** by modifying the **wave velocity** (through the internal fluxes) [3].

The PI controller ⟶ $w_{j,i} = w_{j,i-1} + K_{pj}(e_{j,i} - e_{j,i-1}) + \frac{K_{pj}}{\tau_j} e_i \Delta t$

The NMPC scheme [4]

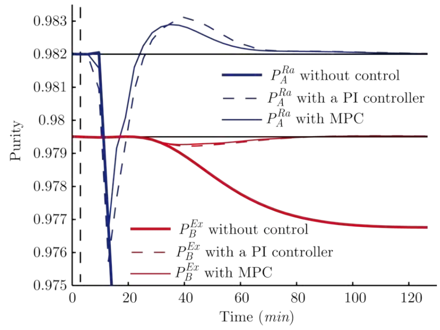
Minimize the error:

$$J = \mathbf{e}_{II}^T \Omega^{II} \mathbf{e}_{II} + \mathbf{e}_{III}^T \Omega^{III} \mathbf{e}_{III}$$

✓ The PI derived in the wave theory is employed

✓ The **parameters** of the controller are computed using the **NMPC scheme**

✓ The POD is employed in the NMPC scheme



Conclusions

We presented a new approach to the control of SMB plants, based on the **combination of NMPC**, **model reduction** and the **wave theory**. The wave theory provide us with the controller form. NMPC (using model reduction) is employed to compute the optimal values of the controller parameters. The **stability properties** of the controller have been tested through a **simulation experiment**.

References
[1] G. Guiochon, S. Golshan-Shirazi, and A.M. Katti. *Fundamentals of Preparative and Nonlinear Chromatography*. Academic Press, New York., 1994.
[2] L. Sirovich. Turbulence and the dynamics of coherent structures. Part I: Coherent structures. *Quaterly of Appl. Math.*, 45(3):561--571, 1987.
[3] H. Schramm, S. Grüner, and A. Kienle. Optimal operation of simulated moving bed chromatographic processes by means of simple feedback control. *Journal of Chromatography A*, 1006(1-2):3--13, 2003.
[4] E. F. Camacho and C. Bordons. *Model Predictive Control*.Springer Verlag, London, UK, 1999.

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